

THE Be SPECTRUM VARIABLE, χ OPHIUCHI
THE SPECTRAL VARIATIONS OF γ
CASSIOPEIAE

A DISSERTATION

SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF DOCTOR OF
PHILOSOPHY IN THE UNIVERSITY OF MICHIGAN

C. H. CLEMINSHAW

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ABSTRACT

χ Ophiuchi shows correlated changes of hydrogen velocity and emission ratio, like other Be spectrum variables. A period of about ten years seems probable. The iron emission shows a similar period, but the helium absorption is out of phase and is more irregular. The helium singlets, λ 4144 and λ 4388, give a mean velocity of 22 km/sec greater than the helium triplets, λ 4121 and λ 4472.

χ Ophiuchi¹ belongs to spectral class Be, which is characterized by emission lines of hydrogen superimposed upon diffuse absorption lines. The spectrum of this star has unusually strong emission lines, which usually appear single on the Ann Arbor spectrograms. Variation in the relative intensity of the violet and red edges of the lines was found by McLaughlin.²

Changes in the emission ratio in other Be stars have been found to be accompanied by changes in radial velocity. This star was found to have a variable velocity, from two spectrograms in 1896 and one in 1902, taken at the Lick Observatory.³ Measurements of the central absorption of $H\gamma$ on these three-prism plates gave -11 km/sec for the first two and $+21$ km/sec for the third. In 1925, four one-prism plates obtained at the Lick Observatory⁴ gave a mean velocity of -11 km/sec. However, this value depends on the lines of several elements in addition to hydrogen.

The object of this investigation is to determine radial velocities from the lines of hydrogen, helium, and ionized iron, on an extensive series of spectrograms, and to correlate the velocities with the changes of the emission lines.

THE OBSERVATIONS

This study is based upon 113 one-prism spectrograms of χ Ophiuchi taken at the Observatory of the University of Michigan. Five plates were obtained in 1913 and the rest during the interval

¹ α 16^h21^m2, δ $-18^{\circ}14'$, mag. 4.9, spectrum B3e.

² *Pub. U. of Michigan Obs.*, 4, 184, 1932.

³ *Lick Obs. Bull.*, 5, 175, 1910.

⁴ *Pub. Lick Obs.*, 16, 240, 1928.

1923-1934, inclusive. The southern declination of the star limits the observing season, so that most of the observations were made during the four months April-July, inclusive.

The hydrogen lines $H\beta$, $H\gamma$, and $H\delta$ were measured on almost all the plates. Three settings were made on each line, one on each edge and one bisecting it. The velocity was obtained by averaging the bisection setting with the mean of the edge settings.

Nine lines of ionized iron are definitely present as emission, but most of them are very poor, and only four have been included in the results. These are $\lambda\lambda$ 4233, 4549, 4556, and 4584. Similarly, of the seven absorption lines of helium which were measured, only four have been used. These are $\lambda\lambda$ 4121, 4144, 4388, and 4472. Because of atmospheric extinction of the violet light, resulting from the low altitude of the star, the helium line λ 4026 was too poorly exposed on most of the plates. The line K of ionized calcium was measured on about one-third of the plates. The values of hydrogen emission ratio are those found by McLaughlin.⁵

TABLE I
NORMAL PLACES OF HYDROGEN VELOCITIES

Epoch	JD 242+*	$H\beta$	$H\gamma$	$H\delta$	Weighted Mean	No. of Plates
1913.4.....	9914*	+ 2	+ 1	+ 7	+ 3	5
1923.3.....	3534	+ 1	+ 4	+12	+ 5	8
1924.4.....	3924	+ 3	+ 4	+ 5	+ 4	11
1925.4.....	4302	- 4	- 2	0	- 2	7
1926.4.....	4668	- 8	- 7	- 9	- 8	8
1928.6.....	5471	-25	-21	-13	-20	1
1929.5.....	5784	- 9	- 6	- 1	- 6	21
1930.5.....	6142	- 6	- 6	+ 2	- 4	15
1931.4.....	6479	- 5	- 1	- 4	- 3	7
1932.4.....	6866	0	0	0	0	7
1933.4.....	7235	+ 3	+ 7	+10	+ 7	17
1934.5.....	7615	+ 3	+ 6	+ 9	+ 6	6

* First three figures are 241 for first entry.

RESULTS OF THE OBSERVATIONS

Tables I-IV give the annual means of the individual lines and of the weighted means. $H\gamma$ was given a weight of 2, and $H\beta$ and $H\delta$

⁵ *Op. cit.*, p. 185.

each a weight of 1. Of the ionized iron lines, $\lambda 4233$ has a weight 2 and the others each 1. The helium lines have been divided into two

TABLE II
NORMAL PLACES OF IRON VELOCITIES

JD 242+*	4233	4549	4556	4584	Weighted Mean	No. of Plates
9914*	+43	+7	-10	+7	+15	5
3534	33	+17	+11	+2	+19	8
3924	22	+3	+25	+13	+16	10
4302	0	-21	-10	-13	-4	6
4668	8	-9	-15	-8	-2	6
5471	15	-5	+13	-35	+1	1
5784	3	-4	+18	-9	+2	20
6142	6	+2	+16	+8	+8	13
6479	20	-9	+15	-17	+5	7
6866	38	-14	+20	+7	+17	7
7235	16	-4	+9	-3	+7	15
7615	+33	+14	+25	+4	+21	6

* First three figures are 241 for first entry.

TABLE III
NORMAL PLACES OF HELIUM VELOCITIES

JD 242+*	TRIPLETS			SINGLET			WEIGHTED MEAN OF 4 LINES	No. OF PLATES
	4121	4472	Weighted Mean	4144	4388	Weighted Mean		
9914*	-39	-8	-11	+11	+20	+17	+3	5
3534	-23	14	16	-4	+5	+2	-7	8
3924	-10	22	18	+3	+2	+2	-8	11
4302	-15	26	22	+9	+1	+3	-9	6
4668	-25	18	19	-9	-7	-8	-14	6
5471	-2	17	12	+40	-26	-4	-8	1
5784	+11	10	4	-1	+6	+4	0	19
6142	-10	13	12	+14	+22	+20	+4	15
6479	+2	12	8	+8	+2	+4	-2	7
6866	-25	11	14	+16	+11	+13	-1	7
7235	-17	26	23	+6	+6	+6	-8	15
7615	-6	-30	-23	+29	+13	+17	-3	6

* First three figures are 241 for first entry.

groups, triplets and singlets. Of the triplets, $\lambda 4472$ has twice the weight of $\lambda 4121$; and of the singlets, $\lambda 4388$ has twice the weight of $\lambda 4144$. Table V gives the widths in angstroms and the emission ratios of the hydrogen lines.

As a check on some of the measurements, it is worth while to include a summary of the results obtained by two other measurers. In 1924, P. A. Smith measured 6 of the 1923 plates and 5 of the 1924

TABLE IV
ANNUAL MEANS OF CALCIUM (K) VELOCITY

JD 242+*	Velocity	No. of Plates	JD 242+	Velocity	No. of Plates
9907*	-23	1	6479	-15	5
3924	10	4	6866	12	3
4302	13	2	7235	13	9
4668	3	2	7581	-19	1
5784	13	9			
6142	-9	6	Mean	-12	

* First three figures are 241 for first entry.

TABLE V
HYDROGEN-LINE WIDTHS AND EMISSION RATIO

JD 242+*	LINE WIDTHS IN ANGSTROMS			EMISSION RATIO
	H β	H γ	H δ	
9914*	3.93	3.00		$R > V$
3534	4.16	2.78	2.82	$V \geq R$
3924	4.56	3.01	3.17	$R = V$
4302	3.66	2.74	3.24	$R = V$
4668	3.59	2.62	2.95	$R \gg V$
5471	2.91	2.48	2.81	$R \geq V$
5784	3.72	2.68	2.82	$R \geq V$
6142	4.16	2.93	2.79	$R \geq V$
6479	4.72	3.05	2.76	$R > V$
6866	4.41	3.13	2.73	$R = V$
7235	4.75	3.13	3.01	$R = V$
7615	3.56	2.56	2.10	$R = V$

* First three figures are 241 for first entry.

plates. In 1926, M. L. Zimmer measured all the plates taken in 1923, 1924, and 1925—a total of 26. The results are given in Table VI. Although the velocities from individual plates scatter more widely, it is reassuring to find the yearly means in substantial agreement.

All the writer's normal places are presented graphically in Figure 1. The emission ratios, and the weighted means of hydrogen, of

helium, and of iron are all represented by dots. The triangles stand for the weighted means of the helium triplets; the circles, for the

TABLE VI
RESULTS OF THREE MEASURERS

Year*	Hydrogen	Iron	Helium Triplets	Helium Singlets	All Helium
1923 (S)....	+6	+18	-13	+9	-2
(Z)....	+8	+23	11	-3	7
(C)...	+5	+19	16	+2	7
1924 (S)....	+7	+20	10	0	5
(Z)....	+3	+11	15	0	8
(C)...	+4	+16	18	+2	8
1925 (Z)....	0	+4	18	-6	12
(C)...	-2	-4	-22	+3	-9

* S, Smith; Z, Zimmer; C, Cleminshaw.

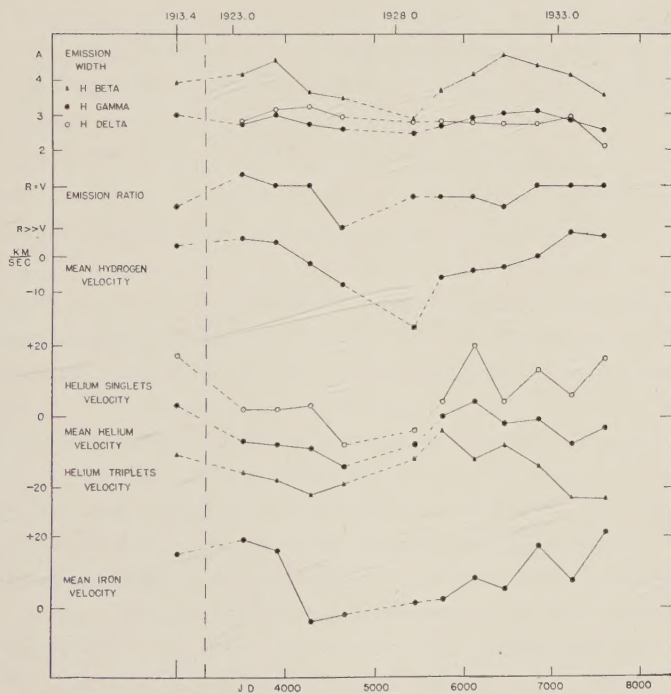


FIG. 1.—Changes in the emission lines and radial velocities of χ Ophiuchi

weighted means of the helium singlets. In the case of the line widths, $H\beta$ is represented by triangles, $H\gamma$ by dots, and $H\delta$ by circles. Connecting lines have been drawn as guides to the eye. The points at JD 5471 have been joined to the other points by broken lines, since they depend on only one plate in 1928. That plate was also measured by Dr. McLaughlin, and the adopted values are the means of his measures and the writer's. The 1913 results have been separated from the rest by a vertical broken line to emphasize the gap of ten years occurring there.

DISCUSSION OF RESULTS

Hydrogen.—There is no doubt of the variation of the hydrogen velocity. A period of about ten years is indicated, with maxima probably occurring in 1923 and 1933. The close agreement of the 1913 velocity with that in 1923 lends a little support to such a period. The 1896 Lick value of -11 km/sec is close to the 1926 value of -8 km/sec. However, the agreement of a few scattered points of possible earlier cycles is not very significant, and it must be understood that the suggested period is only tentative. It is unfortunate that no plates were obtained in 1927 and that only one was obtained in 1928, as a minimum was apparently reached during that interval. The one velocity during that time happens to be consistent with the rest of the curve.

The mean of all the hydrogen velocities is -1 km/sec. There appear to be variations of short period, and an unsuccessful attempt was made to determine this period. The changes are perhaps irregular.

The negative velocities are accompanied by a strengthening of the red emission edges relative to the violet. The 1928 value of the emission ratio should be lower, but these estimates are difficult to make, at best, and a single one is of little weight. The value at JD 6479 is also a little low, but the characteristic behavior of the Be stars is shown in the general correlation of the velocity-curve and the curve of the emission ratio.

The widths of the emission lines, $H\gamma$ and $H\delta$, appear to have been nearly constant, but the $H\beta$ curve looks like the velocity-curve, ex-

cept that the period is shorter. The means of all measures and their numbers are as follows:

Line	Width	Number of Measures
$H\beta$	4.12 Å	107
$H\gamma$	2.87	105
$H\delta$	2.86	73

$H\gamma$ is of the same width as $H\delta$, which is unusual. However, these lines are narrow, and small differences of width are not easily detected. The narrowness of the lines may be attributed to a small inclination of the star's axis of rotation, as was suggested by Struve.⁶ The total intensity of the emission has undergone no appreciable change.

Iron.—The velocity-curve of ionized iron is similar to that of hydrogen, though the variations do not occur simultaneously. The observations are not inconsistent with the suggested ten-year period, though the changes are not as regular as in the case of hydrogen. The mean of all iron velocities is +8 km/sec.

Helium.—The helium velocities are more irregular and are out of phase with those of hydrogen and iron. The most striking fact is the difference in the velocities of the singlets and the triplets. λ 4144 and λ 4388 belong to the same series, diffuse singlets. λ 4121 and λ 4472 are both triplets, though the first belongs to the sharp series and the second to the diffuse series. There is an average difference of 22 km/sec between the two pairs, the triplets having a mean velocity of -15 km/sec and the singlets +7 km/sec. The forms of the two curves are somewhat similar, but the curve for the singlets lags about one year behind the other.

A difference in behavior of these lines was found in β Monocerotis by Dr. M. Alberta Hawes.⁷ The triplets and singlets were out of phase, but no mention was made of any difference in the mean velocities. M. K. Jessup, in an unpublished study of κ Draconis, found a difference in velocity of triplets and singlets in the same direction

⁶ *Aph. J.*, 73, 99, 1931.

⁷ *Pub. Vassar Col. Obs.*, 4, 36, 1934.

and of the same order of magnitude as in χ Ophiuchi. These three instances may indicate that such differences are characteristic of helium in the atmospheres of Be stars. One might well ask whether the cause is to be found in a different location in the atmosphere of the atoms producing the two sets of lines or in actual physical effects of environment upon the atoms in the states involved. Not even a guess can now be hazarded as to the answer to this question.

In this connection, it should be pointed out that Struve⁸ found that the relative intensities of the helium lines are not the same in all stars. His results show that the ratio of intensity of singlet to triplet (4388/4472, 4009/4026, and 4144/4121) is greatest for stars of type B2 and decreases rapidly toward O and slowly toward B9. This behavior of the helium lines was also noted by Marshall.⁹

Calcium.—The K line of ionized calcium is of interstellar origin. The measures of it on 42 plates give a mean velocity of -12 km/sec. The angular distance of χ Ophiuchi from the solar apex at $18^{\text{h}} 5^{\text{m}}$ and $+28^\circ$ is 52° . The component of the solar motion (19 km/sec) in the direction of this star is, therefore, -11.7 km/sec, in excellent agreement with the measured value.

CONCLUSION

χ Ophiuchi takes its place with the other Be spectrum variables in showing correlated changes of hydrogen velocity and emission ratio. It also has other points of resemblance to two Be stars. Agreement between the hydrogen and iron velocity-curves was found in π Camelopardalis;¹⁰ also in that star and in β Piscium¹¹ the helium variations were out of phase with those of hydrogen. These two Be stars are like χ Ophiuchi in having single emission lines. It would seem to be a characteristic of Be stars for helium to be out of phase. The difference in velocity between the helium singlets and triplets should be looked for in other Be stars.

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⁸ *Ap. J.*, **74**, 248, 1931.

⁹ *Pub. U. of Michigan Obs.*, **5**, 162, 1934.

¹⁰ *Pub. Vassar Col. Obs.*, **4**, 41, 1934.

¹¹ *Ibid.*, p. 53.

THE SPECTRAL VARIATIONS OF γ CASSIOPEIAE

C. H. CLEMINSHAW

ABSTRACT

A study of 160 spectrograms of γ Cassiopeiae taken at the University of Michigan Observatory during an interval of 21 years (1914–1935) shows that its behavior is characteristic of the Be spectrum variables. The hydrogen lines show a correlation between changes of radial velocity and variations in the intensity ratio of the violet to the red emission components.

Observations from 1914 through 1928, like those made by R. H. Curtiss from 1911 to 1914, do not show any undoubted variation in velocity or emission ratio. Systematic changes began in 1929, resulting in a slow decrease in the hydrogen emission ratio and in the velocities of hydrogen and ionized iron. This was followed in 1932 by a more rapid increase, the violet component at the end of 1933 being more than twice as strong as the red, and the hydrogen velocity increasing by about 20 km/sec. During part of 1934 the lines were narrow and appeared single; and when both components were visible again at the end of 1934, the red one was nearly three times as strong as the violet, and there was a corresponding decrease of velocity.

The behavior of this star has been irregular, but it has been much like that of another Be star, β Monocerotis.

γ Cassiopeiae¹ was first observed spectroscopically in 1866, when Secchi noticed emission lines in its spectrum. A complete summary of the observations from that date until 1914 is given in the work of Curtiss.² Although the velocity was announced by Hartmann³ as variable, the observations by Curtiss from 1911 to 1914 do not show any undoubted variation. The mean velocity obtained by him from 74 plates was -6.4 km/sec. The Lick observations⁴ give a value of -4.7 km/sec. An emission velocity of -14.4 km/sec and an absorption velocity of $+8.5$ km/sec were found at the Yerkes Observatory.⁵ The latter value, however, depends on only three plates. The velocity adopted in *Moore's General Catalogue of Radial Velocities*⁶ is -6.8 km/sec.

Each of the hydrogen emission lines on the Michigan plates, except $H\beta$, is divided into two components by a central absorption line. In 1932, McLaughlin⁷ suspected a slight change in the intensity ratio of the violet to the red emission component, especially from

¹ α 0^h50^m7, δ $+60^{\circ}11'$, mag. 2.25, spectrum Boe. ³ *A.N.*, **173**, 102, 1906.

² *Pub. U. of Michigan Obs.*, **2**, 1, 1916.

⁴ *Pub. Lick Obs.*, **16**, 11, 1928.

⁵ Frost, Barrett, and Struve, *A.p. J.*, **64**, 11, 31, 1926.

⁶ *Pub. Lick Obs.*, **18**, 7, 1932.

⁷ *Pub. U. of Michigan Obs.*, **4**, 177, 1932.

1915 to 1925. During the latter half of 1932, the violet component became much stronger than the red, and this led to the present attempt to determine whether this star, like other Be stars, has a variable radial velocity. The change in the emission ratio was found independently by Lockyer.⁸

THE OBSERVATIONS

This study is based upon 160 spectrograms of γ Cassiopeiae taken at the Observatory of the University of Michigan between October 24, 1914, and March 23, 1935. All of these were made with the one-prism spectrograph attached to the 37½-inch reflecting telescope, with the exception of 16 plates which were taken with the two-prism spectrograph from August 21, 1927, to July 24, 1928, and one two-prism plate on January 27, 1935.

The hydrogen lines $H\beta$, $H\gamma$, and $H\delta$ were measured on almost all plates; and $H\epsilon$ and $H\zeta$ were measured on a few. $H\beta$ appears as a single emission line and three settings were made on it, one on each edge and one bisecting it. The bisection setting was averaged with the mean of the edge settings to give the adopted "velocity of the emission edges." The other hydrogen lines appear double, and five settings were made on each of them as follows:

1. Violet edge of violet emission component
2. Center of violet emission component
3. Center of central absorption component
4. Center of red emission component
5. Red edge of red emission component

From the mean of the edge settings was derived the velocity of the emission edges, and from the mean of the settings on the emission centers was found the velocity of the emission centers. The third setting gave the velocity of the central absorption. Emission widths were computed from the differences of the edge settings.

Measures were also made, on some of the plates, of several absorption lines of helium and of emission lines of ionized iron. However, these lines are so poor that the results were rejected, except in the case of the ionized iron lines, λ 4233 and λ 4584.

Visual estimates were made, by Dr. McLaughlin, of the ratios of intensity of the violet to the red component at $H\gamma$ and $H\delta$. This is

⁸ *M.N.*, **93**, 362, 619, 1933.

referred to as V/R . He also estimated on many plates the ratio E/C , which is the intensity of the emission line relative to the neighboring continuous spectrum.

TABLE I
NORMAL PLACES OF VELOCITIES IN KM/SEC

JD 242+	ABSORPTION			EMISSION CENTERS			EMISSION EDGES				No. OF PLATES
	$H\gamma$	$H\delta$	Wtd. Mean	$H\gamma$	$H\delta$	Wtd. Mean	$H\beta$	$H\gamma$	$H\delta$	Wtd. Mean	
0447.....	-5	-10	-7	-6	-8	-7	-3	+4	-2	+1	4
0865.....	+13	-6	+7	+11	-8	+5	-6	-1	-15	-6	3
1150.....	-3	+1	-2	-1		-1	-15	0	-15	-7	2
1506.....	-5	-10	-7	-7	-10	-8	-16	-9	-6	-10	2
2280.....	-10	0	-7	-9	+2	-5	-8	-7	-4	-7	9
2682.....	-1	-9	-4	+3	-6	0	-3	+2	-10	-2	4
2955.....	-7	-13	-9	-4	-12	-7	-11	0	0	-3	2
3726.....	+8	+6	+7	+2	+5	+3	+6	-1	0	+1	3
4108.....	-2	-20	-8	0	-15	-5	-2	-9	0	-5	2
4481.....	+3	-8	-1	+1	-4	-1	-2	-1	-5	-2	7
4850.....	-12	-5	-10	-11	-6	-9	-6	-8	-8	-8	15
4900.....	-3	-4	-3	+2	-4	0	-4	-8	-1	-5	12
5155.....	-5	-3	-4	-3	-4	-3	-4	-3	-4	-4	14
5492.....	-9	-6	-8	0	-5	-2	-14	-8		-10	3
5860.....	-15	-20	-17	-14	-14	-14	-6	-14	-10	-11	6
6254.....	-16	-11	-14	-14	-4	-11	-12	-11	-15	-12	4
6639.....		-8	-8		-26	-26	0	-16	-20	-13	2
6912.....	-5	-23	-11	0	-15	-5	-17	0	-18	-9	2
6955.....	-10	-2	-7	-15	-9	-13	-7	-14	-3	-9	2
7007.....	+10	+7	+9	+15	+2	+11	+11	+8	0	+7	2
7028.....	-6	+11	0	-10	+8	-4	-11	-10	-2	-8	3
7045.....	-11	+19	-1	-4	+9	0	-11	-8	+3	-6	5
7126.....	-2	-8	-4	-2	-7	-4	(-14)	-2	-2	-2	5
7228.....	-3	0	-2	-7	-2	-5	(-25)	+3	+3	+3	6
7264.....	+7	-8	+2	+4	-4	+1	(-22)	-5	+2	-3	8
7284.....	+1	+4	+2	-2	+2	-1		-9	+4	-5	4
7345.....	+8	+22	+13	+9	+16	+11		+10	+25	+15	4
7382.....	+11	+15	+12	+2	+10	+5		+3	+14	+7	3
7478.....	+13	+18	+15	+1	+13	+5	(-29)	-9	+2	-5	9
7579.....								-15	-4	-11	2
7713.....								+5	+6	+5	6
7750.....	-34	-39	-36	-23	-26	-24		-6	-14	-9	2
7833.....	-6	-21	-11	-5	-15	-8		-7	-12	-9	3

RESULTS OF THE INVESTIGATION

The observations from the 160 spectrograms have been grouped into 33 normal places in Table I. This gives the velocities of the cen-

tral absorption and of the emission centers for $H\gamma$ and $H\delta$, and the velocity of the emission edges for $H\beta$, $H\gamma$, and $H\delta$. Weighted means are also included, $H\gamma$ having a weight of 2, and $H\beta$ and $H\delta$ each hav-

TABLE II
NORMAL PLACES OF EMISSION RATIOS AND EMISSION WIDTHS

JD 242+	EMISSION RATIO (V/R)			EMISSION WIDTHS IN A			No. OF PLATES
	$H\gamma$	$H\delta$	Wtd. Mean	$H\beta$	$H\gamma$	$H\delta$	
0447.....	1.02	1.00	1.01	4.63	4.24	3.84	4
0865.....	1.20	1.10	1.17	5.00	4.52	4.07	3
1150.....	1.20	1.00	1.13	5.22	3.81	3.49	2
1596.....	1.20	1.00	1.13	5.53	4.40	3.94	2
2280.....	1.00	1.07	1.02	5.34	4.28	4.24	9
2682.....	1.08	1.02	1.06	5.12	4.18	3.83	4
2955.....	0.80	1.00	0.87	5.31	4.10	4.22	2
3726.....	1.00	1.00	1.00	5.03	4.14	4.11	3
4108.....	1.15	1.00	1.10	5.38	4.60	4.53	2
4481.....	1.13	1.06	1.11	5.41	4.58	3.99	7
4850.....	1.05	1.03	1.04	5.91	4.58	4.62	15
4900.....	0.96	1.10	1.01	5.41	4.40	4.19	12
5155.....	1.01	1.03	1.02	4.96	3.95	3.87	14
5492.....	1.03	1.03	1.03	5.19	4.03	4.27	3
5860.....	0.95	1.05	0.98	5.44	4.40	4.24	6
6254.....	0.82	1.13	0.92	5.44	4.04	4.00	4
6639.....	0.85	0.85	0.85	6.09	4.32	3.49	2
6912.....	0.95	1.05	0.98	5.44	4.54	4.08	2
6955.....	1.05	1.30	1.13	5.69	4.86	4.13	2
7007.....	1.20	1.40	1.27	6.06	4.94	4.54	2
7028.....	1.30	1.40	1.33	6.69	5.29	4.93	3
7045.....	1.06	1.20	1.11	6.13	4.80	4.54	5
7126.....	1.48	1.58	1.51	5.22	4.62	4.34	5
7228.....	1.83	2.05	1.90	4.94	4.46	4.40	6
7264.....	2.18	2.03	2.13	6.03	4.60	4.21	8
7284.....	1.90	2.23	2.01		4.68	4.18	4
7345.....	2.50	2.70	2.57		4.46	4.43	4
7382.....	2.37	2.33	2.36		4.40	4.18	3
7478.....	2.30	1.95	2.18	4.47	3.71	3.79	9
7579.....	1.50	2.50	1.83	4.63	3.17	3.11	2
7713.....	1.00	1.00	1.00	3.31	3.03	2.57	6
7750.....	0.33	0.45	0.37		3.41	3.03	2
7833.....	0.38	0.39	0.38		3.50	3.22	3

ing a weight of 1. In the case of the velocity of the emission edges, the weighted means from JD 7126 to the end of the table depend on only $H\gamma$ and $H\delta$, and the values of $H\beta$ have been put in parentheses. The reason for this will be discussed later.

Table II contains the normal places of the emission ratio V/R and the emission widths. Weighted means of V/R are included, but no means of emission widths, since the values vary with wavelength.

The normal places are presented graphically in Figures 1 and 2. Figure 1 shows the line widths in angstroms, the emission ratio

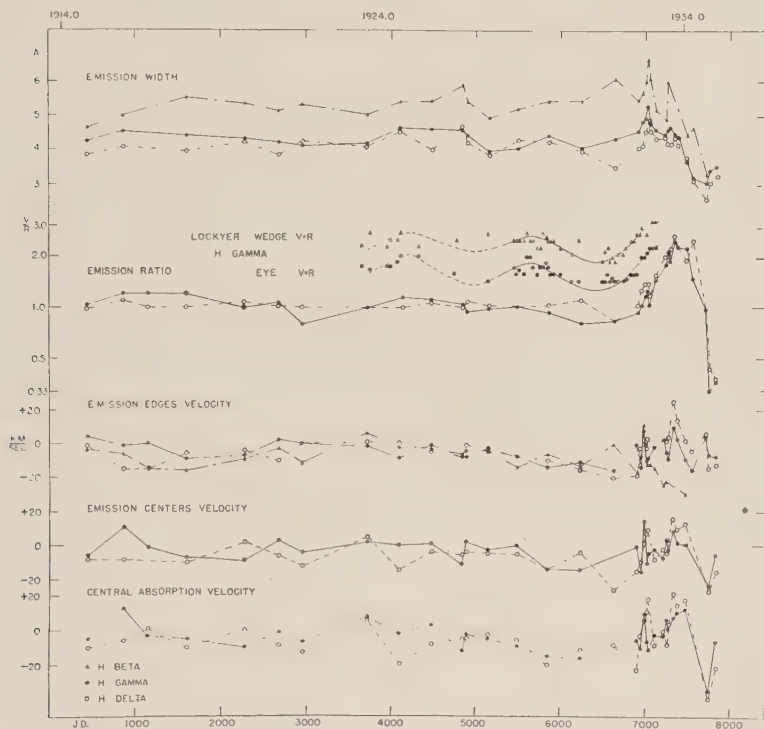


FIG. 1.—Line widths, emission ratios, and radial velocities in γ Cassiopeiae

V/R plotted logarithmically, and the velocities of the individual hydrogen lines. $H\beta$ is represented by triangles, $H\gamma$ by dots, and $H\delta$ by circles. To facilitate following the changes, the points have been connected by straight lines, which are not intended to represent the exact course of the variations.

For comparison, Lockyer's values of the emission ratio for $H\gamma$ have been reproduced from his paper,⁹ in which he tentatively suggested

⁹ *Ibid.*, p. 369.

a period of about four years. The filled triangles represent the "good," and the open triangles the "poor," wedge measures. The dots represent the "good," and the circles the "moderate," eye estimates. The curves have been inverted in order to agree in direction with the writer's curve, but no other change has been made in them. After Figure 1 had been completed, another paper by Lockyer appeared,¹⁰ containing more observations, which have not been

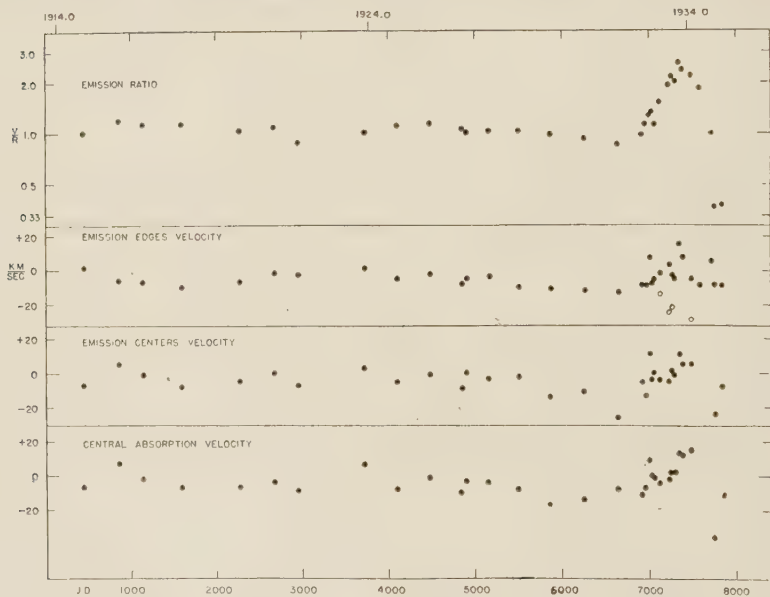


FIG. 2.—Means of hydrogen velocities and of emission ratios in γ Cassiopeiae

reproduced. In this later paper, he abandons the possibility of a four-year period.

Figure 2 represents the weighted means of the hydrogen velocity and of the emission ratio.

DISCUSSION OF RESULTS

In the discussion of the results, the normal places will be referred to by their Julian Day numbers, with the first three figures (242) omitted. First, it is of interest to examine the curves for a periodic variation. The most that can be said is that the velocity curves of

¹⁰ *Ibid.*, 95, 520, 1935.

Figure 2 show very slight indications of a period of about eleven years, with minima near JD 2000 and JD 6000. In this connection, four Yerkes observations¹¹ in 1918 give a mean emission velocity of -21 km/sec. This rather large negative value occurred during a season when no observations were made at Ann Arbor. The epoch for these four plates is JD 1811, near the time of the suggested possible minimum.

However, considering the interval from 1914 through 1928, including the normal place at JD 5492, one might well say that variations of velocity cannot be regarded as definitely established. In particular, there are only three points which depart conspicuously from the mean. Those at JD 0865 (second from the left) and at JD 3726 (middle of diagram) each depend on three plates, and each is influenced by a large positive value of one plate. The point at JD 4108 depends on only two plates, one of which is overexposed.

The weighted mean velocities of the 14 normal places in this interval (1914-1928) are -4.8 km/sec for the absorption, -3.4 km/sec for the emission centers, and -5.0 km/sec for the emission edges, with a probable error of ± 0.8 km/sec for the first two and ± 0.5 km/sec for the third. These are based on 82 plates and are very close to the value of -6.4 km/sec obtained by Curtiss for the years 1911-1914. The constancy of the velocity is confirmed by the emission ratio, which has a mean value of 1.04 for those 14 years. It dropped below unity only once, and it was never greater than 1.2 during that time. The conclusion is that the velocity and emission ratio remained constant from 1911 through 1928, except for possible short-period variations of very small amplitude.

Beginning in 1929, however, there is evidence of a systematic change. The normal place just preceding JD 6000 and the two following it are considerably lower than those in the first large group (1914-1928). The drop of the emission ratio below unity is confirmed by Lockyer's curves, which show the red component a little brighter than the violet. This is accompanied by drops in the velocity-curves, the change in absorption velocity slightly preceding the others.

In 1932 there is a very marked change in the opposite direction. All three emission-ratio-curves agree in showing that the violet com-

¹¹ *Ap. J.*, 64, 31, 1926.

ponent becomes increasingly stronger than the red, and that the velocities become positive. There is an interruption in the rise of the curves; and for that reason the twelve normal places, from JD 6912 to JD 7478, inclusive, have been divided into two groups for comparison. The means of each group, together with the means of the two groups previously discussed, are given in Table III.

If the first group may be considered normal, the second is well below normal, the third slightly above, and the fourth well above. All three velocities and the emission ratio show this variation. The last value of the velocity of the emission edges would be lower if $H\beta$ were included in the mean. However, the four rather large negative values

TABLE III
MEANS OF FOUR GROUPS OF NORMAL PLACES

DATE	HYDROGEN					IRON			
	Abs.	Em. Centers	Em. Edges	V/R	No. of Plates	λ 4233	λ 4584	Mean	No. of Plates
1914-1928.....	- 4.8	- 3.4	- 5.0	1.04	82	- 8	- 7	- 8	24
1929-1931.....	-14.5	-15.0	-11.7	0.94	12	-25	- 2	-13	2
1932.9.....	- 2.3	- 2.4	- 4.5	1.25	19	+ 8	- 6	+ 1	10
1933.7.....	+ 6.9	+ 2.3	+ 0.3	2.16	34	+15	+18	+16	16

of $H\beta$, represented by open circles in Figure 2, are believed to be erroneous, owing to the shift of the center of density of the unresolved double line toward the violet. $H\beta$ is not resolved on the plates, and the increase in the strength of the violet part relative to the red probably causes one to make settings systematically in error in the negative direction.

The velocities derived from absorption, emission centers, and emission edges are in close agreement within each of the first three groups in Table III. In the fourth group, the absorption velocity is about 6 km/sec higher than the mean of the emission velocities. However, it is uncertain whether this difference is real. The red component during this interval was so weak on many of the plates that settings on the center and on the edge of it were difficult to make.

Also included in Table III are the results of the measures of the ionized iron lines, λ 4233 and λ 4584. The values of individual plates

scatter considerably, but they have been divided into the same four intervals of time as the hydrogen lines, and means have been taken. From these it appears that iron undergoes changes similar to those of hydrogen, but possibly of greater amplitude.

In May, 1934, a significant change was noted in the hydrogen lines. $H\gamma$ and $H\delta$, which had always been double emission lines, appeared single on May 21 (J.D. 7579). The central absorption was missing, and it did not appear again until the following November. Unfortunately, no plates were taken during the summer, but six plates in September and October show the lines as single.

This confirms the work of Lockyer,¹² who found that " $H\gamma$, $H\delta$ and $H\epsilon$, during the long period from 1934 March 27 to November 2 exhibited only one strong bright line." His interpretation was that "during those months this bright line, which was the violet component in 1934 February, maintained its intensity, but gradually moved towards the red end of the spectrum, and just at the end of this period, namely, 1934 December 20, reached the position where the red component should appear." He reached this conclusion by placing the negatives film to film and noting a distinct creep of the line from its initial position opposite the violet component (on a negative showing both components) to its final position opposite the red component.

It will be of interest to see if such a change is verified by the measures of the Ann Arbor spectrograms. All the plates from 1914 through 1926, during which interval the velocity and the emission ratio appeared to be constant, were used to find what might be called the "normal" positions of the emission components. For $H\gamma$, the results, in km/sec, were -77 for the violet component and $+75$ for the red. For $H\delta$, the values were, respectively, -88 and $+78$. The two plates of May 21, 1934, give a weighted mean velocity of -11 km/sec for the single emission lines, $H\gamma$ and $H\delta$. The corresponding value for the plates from September 25 to October 13 is $+5$ km/sec. From this it is seen that the center of the single emission line was not near the normal position of either the violet or the red components, and that it shifted only 16 km/sec, or about one-tenth the distance separating them.

From the 1914 to 1926 measures, the doublet separation in km/sec

¹² *M.N.*, **95**, 525, 1935.

is 152 for $H\gamma$ and 166 for $H\delta$. From November 7, 1934, to March 23, 1935, both lines have a separation of 116 km/sec. This decrease has been previously noted by J. F. Heard¹³ on several Yerkes plates of higher dispersion. One of these occurred during the time when the emission was single on the Ann Arbor plates. On September 21, 1934, a doublet separation of 113 km/sec is given for the two $Fe\ II$ lines, $\lambda\ 5169$ and $\lambda\ 5316$. This suggests that possibly the reason for the appearance of the lines as single on Lockyer's objective-prism plates and on the Ann Arbor one-prism plates is their limited resolution.

A possible interpretation of what happened is that the strong violet component and the weak red one approached each other until they were unresolved, then became equal in intensity, then reversed their inequality, and finally moved apart again. For the separation of the components to be smallest when they are equal is typical of several Be spectrum variables, notably 25 Orionis¹⁴ and φ Persei.¹⁵ With this interpretation, γ Cassiopeiae has done what other Be stars do, and its behavior seems unusual only because of the more conspicuous change in the width of the lines.

There appears to have been no definite variation in the widths of the emission lines until 1932. At that time there was a marked increase in all three line widths, which reached their maxima when the emission ratio reached its first peak. Then they decreased while the emission ratio went up to its maximum value, though one point on the $H\beta$ curve seems to indicate that it reached a secondary peak. The widths continued their decrease until October, 1934, when they reached minimum values of 3.31 Å for $H\beta$, 3.03 Å for $H\gamma$, and 2.57 Å for $H\delta$. These may be contrasted with the mean widths of the lines, as found by Curtiss¹⁶ and by the writer, as follows:

	$H\beta$	$H\gamma$	$H\delta$
1911-1914.....	5.06 Å	4.40 Å	3.94 Å
1914-1931.....	5.35	4.30	4.14

¹³ *Ap. J.*, **81**, 341, 1935.

¹⁴ Helen W. Dodson, dissertation, University of Michigan, unpublished.

¹⁵ H. F. Schiefer, dissertation, University of Michigan, unpublished.

¹⁶ *Pub. U. of Michigan Obs.*, **2**, 1, 1916.

The ratio of the emission to the continuous spectrum does not show definite variations. When the components are very unequal, there is an appearance of a change; but this is probably not real, for the weakening of one component nearly, if not quite, balances the strengthening of the other. The definite determination of changes of intensity of emission would require measures on a microphotometer, of plates of higher dispersion than those available.

The behavior of this star is characteristic of the Be spectrum variables, as is shown by the correlation of velocities with emission changes. Though the variations appear to be irregular, a striking similarity is shown in the curves of the hydrogen velocities and of the emission-ratios of β Monocerotis, which was studied by Dr. Hawes.¹⁷ In that case, the emission ratio, V/R , of $H\beta$ remained

TABLE IV

	Abs. Vel.	Em. Vel.	Log V/R
	(km/sec)	(km/sec)	
π Aquarii.....	120	90	0.78
25 Orionis.....	130	110	.64
β Monocerotis.....	50	75	.60
γ Cassiopeiae.....	47	32	0.88

equal to unity for about 15 years, then dropped below unity for several years, and finally increased rather suddenly to a value greater than 2. Corresponding changes occurred in the velocity, though it decreased sooner than did V/R . On the basis of the mechanism of Be stars suggested by McLaughlin,¹⁸ it would appear that in the cases of both γ Cassiopeiae and β Monocerotis, the stellar atmosphere remained quiescent for a considerable time, then slowly expanded, and then rapidly collapsed. The changes in γ Cassiopeiae during 1934 would indicate another expansion.

The range in velocity is not as great as might be expected from the changes in the emission ratio. Table IV gives a comparison of the ranges, in round numbers, of absorption and emission velocities and

¹⁷ *Pub. Vassar Coll. Obs.*, **4**, 36, 1934.

¹⁸ *Proc. Nat. Acad.*, **19**, 44, 1933.

of $\log V/R$ of $H\gamma$ in π Aquarii,¹⁹ 25 Orionis,²⁰ β Monocerotis,²¹ and γ Cassiopeiae. Of these four stars, γ Cassiopeiae has the smallest range of velocity and the largest range of emission ratio.

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¹⁹ *Pub. U. of Michigan Obs.*, **4**, 44, 1931.

²⁰ H. W. Dodson, *op. cit.*

²¹ *Pub. Vassar Coll. Obs.*, **4**, 31, 1934.

